Chapter 3 probability الاحتمالات :

تجارب
* Experiment

① Deterministic محددة

② Random عشوائية

* The sample space
(الفضاء العيني)

Ω : defined as the set of all possible outcomes

جميع نتائج التجربة العشوائية

Example Find the sample space for the following:

① Tossing (رمي) a fair (عملة نقدية) coin 1 - time

$\Omega : \{H, T\}$ $\quad 2^1$

② Tossing a fair coin 2-times

$$\Omega : \left\{ \begin{matrix} (H, H), & (H, T) \\ (T, H), & (T, T) \end{matrix} \right\} \quad 2^2$$

③ Tossing a fair coin 3-times

$$\Omega : \left\{ \begin{matrix} (H, H, H), & (H, H, T), & (H, T, H), & (H, T, T) \\ (T, H, H), & (T, H, T), & (T, T, H), & (T, T, T) \end{matrix} \right.$$

$$2^3$$

<u>NOTE</u> when tossing a fair coin k-times, So the number of elements in a sample space is 2^k

1) Throwing a fair dice 1-time
حجر النرد

$\Omega : \{1, 2, 3, 4, 5, 6\}$ $\quad 6^1$

2) Throwing a fair dice 2-times

6^2

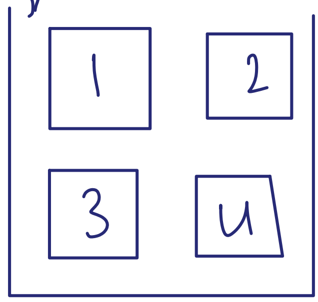
	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

NOTE when throwing a fair dice k times, so the number of elements in a sample space is 6^k

Example 2 - cards are drawn from the Box that contained 4 - cards numbered as (1 - 4). Find the sample space when:

① The drawn was <u>with replacement</u> مع ارجاع

	1	2	3	4
1	(1,1)	(1,2)	(1,3)	(1,4)
2	(2,1)	(2,2)	(2,3)	(2,4)
3	(3,1)	(3,2)	(3,3)	(3,4)
4	(4,1)	(4,2)	(4,3)	(4,4)



② The drawn was <u>without Replac.</u> بدون ارجاع

	1	2	3	4
1	X	(1,2)	(1,3)	(1,4)
2	(2,1)	X	(2,3)	(2,4)
3	(3,1)	(3,2)	X	(3,4)
4	(4,1)	(4,2)	(4,3)	X

NOTE

☆ إذا السؤال ما حدد مع إرجاع أو بدون إرجاع

نعتمد " بدون إرجاع "

☆ The Event   الحادث

☆ أي مجموعة من عناصر الفضاء العيني

① Simple event : Consist of 1 element of sample space
الحادث البسيط

② Comosite " combined " event : Consist of more than 1 element of sample space
الحادث المركب

③ Certain event : Consist of all elements of sample space
الحادث الأكيد

4) Impossible event : Consist of no elements of sample space

الحدث المستحيل

Example Throwing a dice 1-time, define

$\Omega : \{1, 2, 3, 4, 5, 6\}$

A: {getting a number divisible by 5}

$A : \{5\}$ ⟶ Simple event

عدد أولي

B: {getting a prime number}

عدد أولي : يقبل القسمة على نفسه و على الواحد فقط

عدا رقم (1)

$B : \{2, 3, 5\}$ ⟶ Composite event

C: getting a number less than 7

$C : \{1, 2, 3, 4, 5, 6\}$ ⟶ Certain event

D: getting a number more than 6

D: $\{\ \} = \phi$ $\longrightarrow$ impossible event

The probability of events

احتمال الحوادث

$$P(A) = \frac{n(A)}{n(\Omega)} = \frac{\text{عدد عناصر الحادث } A}{\text{عدد عناصر الفضاء العيني}}$$

Example In previous question:

1. $P(A) = \frac{1}{6}$

2. $P(B) = \frac{3}{6} = \frac{1}{2}$

3. $P(C) = \frac{6}{6} = 1$

4. $P(D) = \frac{0}{6} = 0$

NOTES

~~مهم~~

1. Certain events probability = 1
2. impossible events probability = 0
3. $P(\Omega) = 1$
4. $P(\phi) = 0$

مهم

5. $0 \le P(A) \le 1$

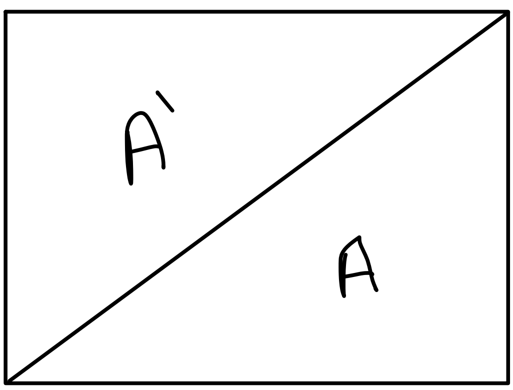
Rules of probability

A: الاحتمال الأول
B: الاحتمال الثاني

قوانين الاحتمالات

intersection	$\cap$	تقاطع
union	$\cup$	اتحاد

$= 1$

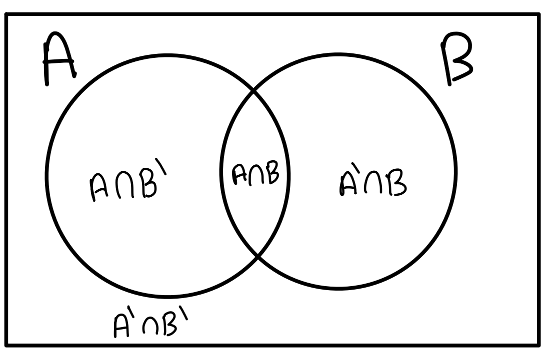


$$P(A) + \underbrace{P(A')} = 1$$

1. $P(A') = 1 - P(A)$

2. $P(A \cap B') = P(A) - P(A \cap B)$

$P(A' \cap B) = P(B) - P(A \cap B)$



3) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
"Addition Rule"

4) $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$
$P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$
"De morgan's Law"

5) A and B are mutually exclusive "disjoint"
if : 1) $P(A \cap B) = 0$
and
2) $P(A \cup B) = P(A) + P(B)$

6) A and B are independent if
$P(A \cap B) = P(A) \cdot P(B)$

NOTE A & B & C independent
$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

"multiplication Rule"

7) $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

given

if

"Conditional probability"

NOTES

1) $P(A|B)$ and A & B are independen[t]

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot \cancel{P(B)}}{\cancel{P(B)}} = P(A)$$

$$P(A|B) \equiv P(A)$$

$$P(A|B') = P(A)$$

2) $P(A \cap B) \equiv P(B \cap A)$ A and B

3) $P(A \cup B) \equiv P(B \cup A)$ A or B

Example If $P(A) = 0.8$, $P(B) = 0.7$
$P(A \cap B) = 0.6$ find:

i) $P(A') = 1 - P(A) = 1 - 0.8 = 0.2$

ii) $P(B') = 1 - P(B) = 1 - 0.7 = 0.3$

iii) $P(A \cap B') = P(A) - P(A \cap B)$
$= 0.8 - 0.6 = 0.2$

iv) $P(A' \cap B) = P(B) - P(A \cap B)$
$= 0.7 - 0.6 = 0.1$

v) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.8 + 0.7 - 0.6 = 0.9$

vi) $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$
$= 1 - 0.9$
$= 0.1$

vii) $P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$
$= 1 - 0.6 = 0.4$

علاوة

viii) $P(A' \cup B) = P(A') + P(B) - P(A' \cap B)$

$= 0.2 + 0.7 - 0.1 = 0.8$

ix) $P(A \cup B') = P(A) + P(B') - P(A \cap B')$

$= 0.8 + 0.3 - 0.2 = 0.9$

x) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.6}{0.7} = \dfrac{6}{7}$

xi) $P(A'|B) = \dfrac{P(A' \cap B)}{P(B)} = \dfrac{0.1}{0.7} = \dfrac{1}{7}$

xii) $P(A|B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{0.2}{0.3} = \dfrac{2}{3}$

xiii) $P(A'|B') = \dfrac{P(A' \cap B')}{P(B')} = \dfrac{0.1}{0.3} = \dfrac{1}{3}$

xiv) $P(A|B)' = 1 - P(A|B) = 1 - \dfrac{6}{7}$

$= \dfrac{1}{7}$

NOTE $P(A|B)' = P(A'|B)$

Example If $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cup B) = 0.8$ are A & B mutually exclusive? independent? neither?

Mutually exclusive

$$P(A \cup B) \stackrel{?}{=} P(A) + P(B)$$

$$0.8 \stackrel{?}{=} 0.6 + 0.5$$

$$0.8 \stackrel{?}{=} 1.1$$

(Not ME)

independent

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.8 = 0.5 + 0.6 - P(A \cap B)$$

$$0.8 = 1.1 - P(A \cap B)$$

$$P(A \cap B) = 1.1 - 0.8$$

$$\boxed{P(A \cap B) = 0.3}$$

$$0.3 \stackrel{?}{=} 0.5 * 0.6$$

$$0.3 = 0.3 \quad ✓$$

So A & B are independent events

Example: If A and B are independent events such that $P(A) = 2 * P(B)$ and $P(A \cup B) = 0.8$ then find $P(A)$?

الحل

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A) = 2 * P(B)$$

$$P(A \cup B) = 0.8$$

$$0.8 = P(A) + P(B) - P(A \cap B)$$

$$0.8 = P(A) + P(B) - P(A) \cdot P(B)$$

$$0.8 = 2 * P(B) + P(B) - 2P(B) * P(B)$$

$$\boxed{X = P(B)}$$

$$0.8 = 2X + X - 2X^2$$

$$2X^2 - 3X + 0.8 = 0$$

$X = 1.53$ ✗ $\quad$ $X = 0.34$ ✓ $\quad$ $\boxed{P(B) = 0.34}$

$$P(A) = 2 * 0.34$$

$$= 0.68$$

* Relative Risk (RR)

خطر

Risk factor
عامل خطر

"Risk ratio"

متعرض

$$RR = \frac{P(B|A)}{P(B|A')} = \frac{\text{Risk for exposure group}}{\text{Risk for unexposure group}}$$

< 1 $= 1$ > 1

* Some interpretations

$\Rightarrow RR = 1.5$ exposure group risk is higher than unexposure groupe in about 0.50

$\Rightarrow RR = 3$ exposure group risk is higher than unexposure groupe in about 3 times

$\Rightarrow RR = 0.8$ exposure group risk is less than unexposure groupe in about 0.20

NOTE If A and B are independent then the RR is 1

توضيح

$$RR = \frac{P(B|A)}{P(B|A')} = \frac{\cancel{P(B)}}{\cancel{P(B)}} = 1$$

Example A study enrolls a 100 smokers and 100 non-smokers. They are followed for next years for developing lung CA. 30 of smokers and 10 of Non-Smokers developed lung CA. Calculate the RR?

اكا

	Smokers	non-Smokers
	100	100
CA	30	10

$$RR = \frac{P(B|A)}{P(B|A')} = \frac{30/100}{10/100} = 3$$

Example If 1 in 10 people exposed to a substance gets sick. If 1 in 100 people who are not exposed get sick. find the RR?

	تعرضوا	لم يتعرضوا
	10	100
Sick	1	1

$$RR = \frac{1/10}{1/100} = 10$$

Example suppose we want to know if exercise affects the risk of developing some disease we collect data and find that 28% of people who exercise regularly develop this disease while 50% of people who do not exercise regularly develop this disease, find the RR?

$$RR = \frac{0.28}{0.50} = 0.56$$

Example Suppose we want to know if some new studying program affects the ability of students to pass a particular exam. we collect a data and find that 40% of students who use the new studying program pass the exam while 40% of students who do not use the studying program also pass the exam, calculate the RR?

$$RR = \frac{0.40}{0.40} = 1$$

Example suppose 50 basketball players use a new training program, and 50 players use an old training program. At the end of the program we test each player to see if they pass a certain skills test. Find RR?

	passed	failed	
New program	34	16	= 50
old program	39	11	= 50

(1) $RR = \dfrac{34/50}{39/50} = 0.872$

B: pass

A: new program

(2) $RR = \dfrac{P(B|A)}{P(B|A')} = \dfrac{\frac{P(B \cap A)}{P(A)}}{\frac{P(B \cap A')}{P(A')}}$

$= \dfrac{\frac{34}{34+16}}{\frac{39}{39+11}} = \dfrac{34/50}{39/50}$

$= 0.872$

Example Suppose that among 100.000 women with negative mammograms 20 will be diagnosed with breast CA within years, whereas 1 women in 10 with positive mammograms will be diagnosed by breast CA within years

find RR ?

$P(B/A') = 0.002$

$P(B/A) = 0.1$

	(+)	(−)
	10	100.000
Breast CA	1	20

$$RR = \frac{1/10}{20/100.000} = 500$$

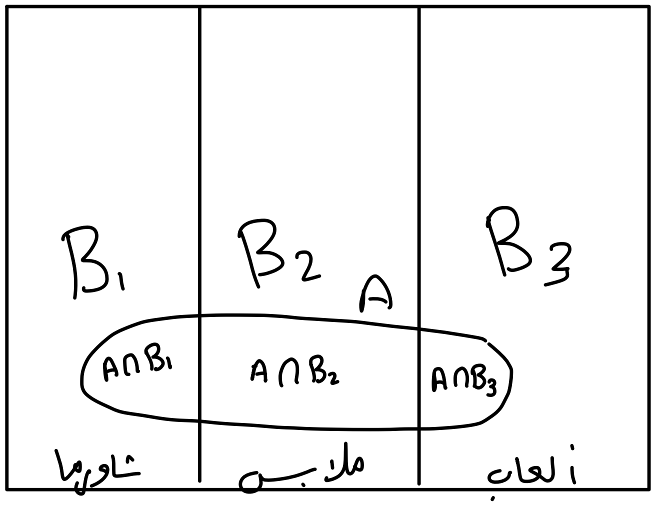
* Total probabity

* mutually exclusive events

A & B

$\Rightarrow$ حدوث A $\Rightarrow$ عدم حدوث B

$\Omega = 1$

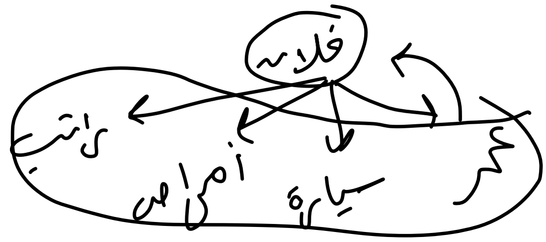


$$P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3)$$

$$\frac{P(A|B_1)}{1} = \frac{P(A \cap B_1)}{P(B_1)} \Rightarrow \boxed{P(A \cap B_1) = P(A|B_1) * P(B_1)}$$

$$P(A) = P(A|B_1) * P(B_1) + P(A|B_2) * P(B_2) + P(A|B_3) * P(B_3)$$

$$\boxed{P(A) = \sum P(A|B_n) * P(B_n)}$$

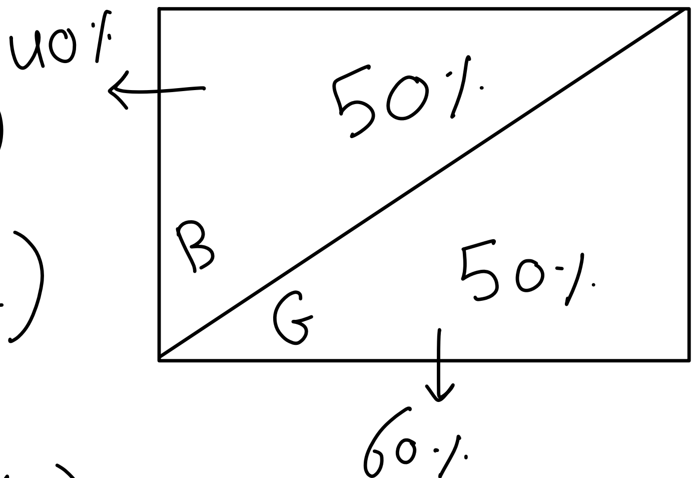


$\Rightarrow$ insurance

$\Rightarrow$ disease

Example If 40% of Boys opted for maths and 60% of Girls opted for maths, then what is the probability that math is chosen if half of class's population is girls?

$$P(\text{maths}) = P(\text{maths} \cap B) + P(\text{maths} \cap G)$$



$$= P(\text{math} \mid B) * P(B) + P(\text{math} \mid G) * P(G)$$

$$= 0.40 * 0.50 + 0.60 * 0.50$$

$$= 0.50$$

Example Company A produces 10% defective products, Company B produces 20% defective products and Company C produces 5% defective products. If choosing a company is an equally likely event, find the probability that the product chosen is defective?

A	B	C
10% D	20% D	5% D
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

$$P(\text{defective}) =$$

$$P(D \cap A) + P(D \cap B) + P(D \cap C)$$

$$= P(D|A) * P(A) + P(D|B) * P(B) + P(D|C) * P(C)$$

$$= 0.10 * \frac{1}{3} + 0.20 * \frac{1}{3} + 0.05 * \frac{1}{3}$$

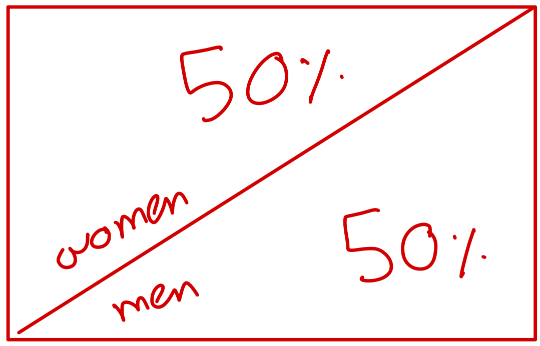
$$= 0.12$$

Example Suppose 5 men out of 100 and 10 women out of 250 are color Blind (عمى الوان), then find the total probability of color blind people?

(Assume that both men and women are equally in number)

$$P(CB) = P(CB \cap W) + P(CB \cap m)$$

$\frac{10}{250}$



5%

$$= P(CB|W) * P(W) + P(CB|m) * P(m)$$

$$= \frac{10}{250} * 0.50 + \frac{5}{100} * 0.50$$

$$= 0.045$$

Example: we are planning a 5 year study of cataract in a population of 5000 people 60 years old and older. we know that:

المي البيضا/الزرق

A_1 : {ages 60 – 64} A_2 : {Ages 65 – 69}

$A_3: \{\text{ages } 70-74\}$ $A_4: \{\text{Ages } 75+\}$

what is the probability of event B which is defined as the probability of developing cataract in the next 5 years, given:

$P(A_1) = 0.45$

$P(A_2) = 0.28$

$P(A_3) = 0.20$

$P(A_4) = 0.07$

$P(B|A_1) = 0.024$

$P(B|A_2) = 0.046$

$P(B|A_3) = 0.088$

$P(B|A_4) = 0.153$

حل

$$P(B) = P(B \cap A_1) + P(B \cap A_2) + P(B \cap A_3) + P(B \cap A_4)$$

$$P(B) = P(B|A_1) * P(A_1) + P(B|A_2) * P(A_2) + P(B|A_3) * P(A_3) + P(B|A_4) * P(A_4)$$

$$= 0.024 * 0.45 + 0.046 * 0.28 + 0.088 * 0.2 + 0.153 * 0.07$$

$= 0.05$

$\therefore$ $\therefore$

Example Suppose 2 doctors A & B, test all patients coming into clinic for syphilis. Let us define the following 2 events:

A+ (doctor A makes a positive diagnosis)

B+ (doctor B makes a positive diagnosis)

$$P(A+) = 0.10, \quad P(B+) = 0.17, \quad P(A+ \cap B+) = 0.08$$

Answer the following:

a) find the conditional probability that doctor B makes a positive diagnosis givin that doctor A makes a positive diagnosis?

$$P(B+ \mid A+) = \frac{P(B+ \cap A+)}{P(A+)} = \frac{0.08}{0.10}$$

$$= 0.8$$

اول

B) what is the conditional probability that doctor B makes a positive diagnosis given that doctor A makes a negative diagnosis?

$$P(B+ \mid A'+) = \frac{P(B+ \cap A'+)}{P(A'+)} = \frac{0.17 - 0.08}{0.90} = 0.1$$

$$P(B+ \cap A'+) = P(B+) - P(A+ \cap B+)$$

c) what is the RR of B+ given A+?

$$RR = \frac{P(B+ \mid A+)}{P(B+ \mid A'+)} = \frac{0.8}{0.1} = 8$$

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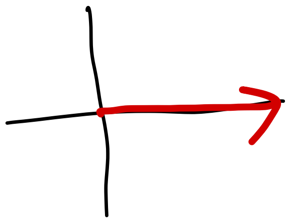
* Baye's Rule and screening tests

		الحقيقة (+)	الحقيقة (−)
Test	(+)	True positive	false positive
Test	(−)	false negative	True negative

① positive predictive value

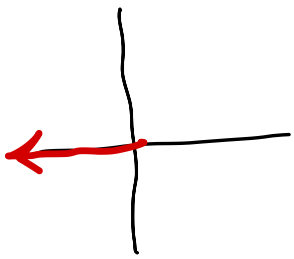
$$PPV \equiv PV+ = \frac{TP}{TP+FP}$$



$$= P(\text{disease} \mid +)$$

② Negative predictive value

$$NPV \equiv PV(-) = \frac{TN}{TN+FN}$$



$$= P(\text{no disease} \mid (-))$$

③ sensitivity $= \dfrac{TP}{FN+TP}$

$$= P((+) \mid \text{disease})$$

④ specifity $= \dfrac{TN}{TN+FP}$

$$P((-) \mid \text{no disease})$$

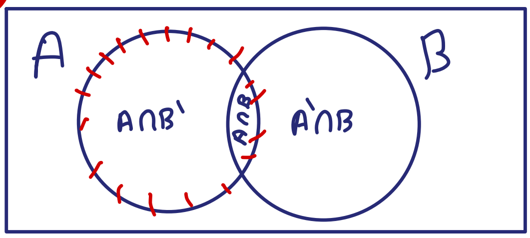
* Baye's Rule

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(B \mid A) = \frac{P(B \cap A)}{P(A)}$$

$$\boxed{P(B \cap A) = P(A \cap B)}$$

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A \mid B) * P(B)}{P(A \cap B) + P(A \cap B')}$$

$$P(B \mid A) = \frac{P(A \mid B) * P(B)}{P(A \mid B) * P(B) + P(A \mid B') * P(B')}$$



Example The table below shows the results from looking at the diagnostic accuracy of a new rapid test for HIV in 100.000

subjects, The rows of the table represent the test Result and the columns the true disease status

	HIV(+)	HIV(−)	Total
Test (+)	378	397	775
Test (−)	2	98823	98825
Total	380	99220	100000

① find PV(+) (PPV)?

$$PPV = P(\text{disease} \mid (+)) = \frac{P(\text{disease} \cap (+))}{P(+)}$$

$$= \frac{378}{775}$$

② find NPV (PV(−))?

$$NPV = P(\text{no disease} \mid (-)) = \frac{P(\text{no} \cap -)}{P(-)}$$

$$= \frac{98823}{98825}$$

(3) find Sensitivity ?

$$P((+) \mid \text{disease}) = \frac{P(+ \cap \text{disease})}{P(\text{disease})} = \frac{378}{380}$$

(4) find the specifity ?

$$P((-) \mid \text{no disease}) = \frac{P(- \cap \text{no})}{P(\text{no})} = \frac{98823}{99220}$$

Example In a hospital, If you know that probability to have certain disease is 0.15 and the probability to have a positive test if you have the disease is 0.95 and a negative test if you don't have the disease is 0.93, and a positive test if you don't have the disease is 0.7

find $PV(+)$?

$$P(B') = 1 - P(B)$$
$$= 1 - 0.15$$
$$= 0.85$$

$$P(D) = 0.15$$

$P(A|B)$ $\quad P(\overset{A}{+} | \overset{B}{D}) = 0.95 \longrightarrow$ sensitivity

$P(A'|B')$ $\quad P(- | D') = 0.93 \longrightarrow$ specifity

$P(A|B')$ $\quad P(+ | D') = 0.7 \longrightarrow$ false positive

A: (+)

B: disease

$$PV(+) = P(D | +)$$

$$P(B|A) = \frac{P(A|B) * P(B)}{P(A|B) * P(B) + P(A|B') * P(B')}$$

$$= \frac{0.95 * 0.15}{0.95 * 0.15 + 0.7 * 0.85}$$

Example Suppose 84% of hypertensive and 23% of normotensive are classified by Automated blood pressure machine as hypertensive what are the (PV+) and (PV−) of the machine if you know that 20% of the adult population have the disease?

ABP

84% HTN D — 23% normotensive D'

$P(+ \mid D')$

$P(\overset{A}{+} \mid \overset{B}{D}) = 0.84 \longrightarrow$ sensitivity

$P(\overset{A}{+} \mid \overset{B'}{D'}) = 0.23$

$P(A' \mid B') = 1 - P(A \mid B') = 1 - 0.23 = 0.77$

A: (+)

B: D

$P(\underset{B}{D}) = 0.20$

$\boxed{\underline{\text{NOTE}} = P(A' \mid B') = 1 - P(A \mid B')}$

$$PV(+) = P(B|A) = \frac{P(A|B) * P(B)}{P(A|B) * P(B) + P(A|B') * P(B')}$$

$$= \frac{0.84 * 0.20}{0.84 * 0.20 + 0.23 * 0.80}$$

$$= 0.48$$

$$PV(-) = P(B'|A') = \frac{P(A'|B') * P(B')}{P(A'|B') * P(B') + P(A'|B) * P(B)}$$

$$= \frac{0.77 * 0.80}{0.77 * 0.80 + 0.16 * 0.20}$$

$$= 0.95$$

NOTE $P(A'|B) = P(A|B)' = 1 - P(A|B)$

Example A man is known to speak the truth 2 out of 3 times. He throws a dice and reports that the number obtained is 4. Find probability that actually the number obtained is 4?

$\Omega : \{1, 2, 3, 4, 5, 6\}$

$P(4) = \frac{1}{6}$

حل

$$P\left(\overset{A}{\underset{4}{\text{سکه}}} \,\middle|\, \overset{B}{4}\right) = \frac{2}{3}$$

$$P\left(\overset{A}{\underset{4}{\text{سکه}}} \,\middle|\, \overset{B'}{4'}\right) = \frac{1}{3}$$

$$P(4) = \frac{1}{6}$$

$$P\left(\overset{B}{4} \,\middle|\, \overset{A}{\underset{4}{\text{سکه}}}\right) = \frac{P(A|B) * P(B)}{P(A|B) * P(B) + P(A|B') * P(B')}$$

$$= \frac{\frac{2}{3} * \frac{1}{6}}{\frac{2}{3} * \frac{1}{6} + \frac{1}{3} * \frac{5}{6}} = \frac{2}{7}$$

Example suppose 50 men out of 1000 men and 25 women out of 1000 are orators (خطباء). An orator is chosen at random. Find the probability that a male person is selected?

$$P(\overset{A}{\text{orator}} \mid \overset{B'}{\text{man}}) = \frac{50}{1000}$$

$$P(\overset{A}{\text{orator}} \mid \overset{B'}{\text{women}}) = \frac{25}{1000}$$

$$P(\overset{B}{\text{man}} \mid \overset{A}{\text{orator}}) = \frac{P(A|B) * P(B)}{P(A|B) * P(B) + P(A|B') * P(B')}$$

$$= \frac{\frac{50}{1000} * \frac{1}{2}}{\frac{50}{1000} * \frac{1}{2} + \frac{25}{1000} * \frac{1}{2}} = \_\_\_$$

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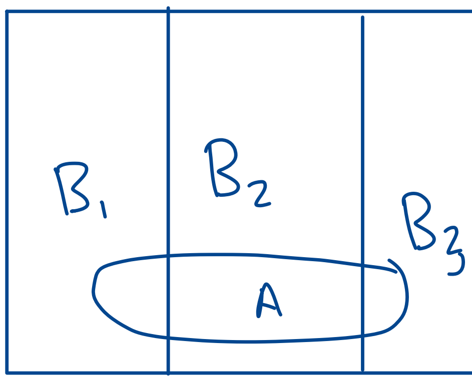
* Generalized Baye's Rule

$$P(A) = \sum P(A|B_n) * P(B_n)$$



$$P(B_n|A) = \frac{P(A|B_n) * P(B_n)}{P(A)} = \frac{P(A|B_n) * P(B_n)}{\sum P(A|B_n) * P(B_n)}$$

Example Three factories A, B and C of an electrical bulb produce Respectively 35% 35% and 30% of total product. Approximately 1.5%, 1% and 2% of the bulbs produced by these factories are known to be defective. IF a randomly selected bulb manufactured by a company was found to be defective, what is the probability that the bulb was manufacters in factory A?

$$P(A) = 0.35 \quad P(B) = 0.35$$

$$P(C) = 0.30$$

$$P(\overset{A}{D} \mid \overset{1}{A}) = 0.015$$

$$P(\overset{A}{D} \mid \overset{2}{B}) = 0.01 \qquad P(\overset{A}{D} \mid \overset{3}{C}) = 0.02$$

$$P(A|D) = \frac{P(D|A) * P(A)}{P(D|A) * P(A) + P(D|B) * P(B) + P(D|C) * P(C)}$$

$$= \frac{0.015 * 0.35}{0.015 * 0.35 + 0.01 * 0.35 + 0.02 * 0.30}$$

$$= 0.356$$