Estimation 6

Solved Problems

Question (1)

Suppose we have a sample of five values of hemoglobin A1c (HgbA1c) obtained from a single diabetic patient. HgbA1c is a serum measure often used to monitor compliance among diabetic patients. The values are 8.5%, 9.3%, 7.9%, 9.2%, and 10.3%. Answer the following:

(a) What is the standard deviation for this sample?

(b) What is the standard error for this sample?

Solution

(a) Sample Standard Deviation (S)

(1) Calculate the sample mean ($\bar{x}$) for the random sample of the five values of hemoglobin A1c (HgbA1c) as follows:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{5} x_i}{5} = \frac{45.2}{5} = 9.04\ \%$$

(2) Calculate the value of the sample variance (S^2) as follows:

$$S^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} = \frac{\sum_{i=1}^{5}(x_i - \bar{x})^2}{5-1}$$

$$= \frac{(8.5-9.04)^2+(9.3-9.04)^2+(7.9-9.04)^2+(9.2-9.04)^2+(10.3-9.04)^2}{4}$$

$$= \frac{0.2916 + 0.0676 + 1.2996 + 0.0256 + 1.5876}{4} = \frac{3.272}{4} = 0.818\ \%$$

(3) Calculate the value of the sample standard deviation (S) as follows:

$$S = \sqrt{S^2} = \sqrt{0.818} = 0.90443\ \%$$

(a) Sample Standard Error (se)

$$se = \frac{s}{\sqrt{n}} = \frac{0.90443}{\sqrt{5}} = 0.40447\ \%$$

Question (2)

Suppose it is known that in a certain large human population cranial length is approximately normally distributed with a mean of 185.6 mm and a standard deviation of 12.7 mm. What is the probability that a random sample of size 10 from this population will have a mean:

(a) less than or equal to 190 mm?

(b) greater than 190 mm?

(c) between 180 mm and 190 mm?

(d) equal to 190 mm?

Solution

We have X = cranial length$\sim N(185.6, (12.7)^2)$, then the sampling distribution of the sample mean ($\overline{\mathrm{X}}$) is exactly normal and it is given as follows:

$$\overline{\mathrm{X}} \sim N\left(\mu_{\bar{X}} = \mu = 185.6\, , \sigma_{\bar{X}}^2 = \frac{\sigma^2}{n} = \frac{(12.7)^2}{10} = 16.129\right)$$

→ $\overline{\mathrm{X}} \sim N(185.6\, , 16.129)$ is the Sampling Distribution of $\overline{\mathrm{X}}$ exactly

with population standard deviation $\sigma_{\bar{X}} = \sqrt{\sigma_{\bar{X}}^2} = \frac{\sigma}{\sqrt{n}} = \sqrt{16.129} = 4.016$

(a) We need to find $P(\bar{X} \le 190)$ as follows:

$$P(\bar{X} \le 190) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \le \frac{190-185.6}{4.016}\right)$$
$$= \phi(1.10)$$
$$= 0.8643 \quad \text{From Column A in Table 3 of the Appendix}$$

Conclusion

Thus 86.43% of the samples of size 10 would be expected to have a mean of cranial length less than or equal to 190 mm.

(b) We need to find $P(\bar{X} > 190)$ as follows:

$$P(\bar{X} > 190) = 1 - P(\bar{X} \le 190)$$
$$= 1 - P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \le \frac{190-185.6}{4.016}\right)$$
$$= 1 - \phi(1.10)$$
$$= 1 - 0.8643 \quad \text{From Column A in Table 3 of the Appendix}$$
$$= 0.1357$$

Conclusion

Thus 13.57% of the samples of size 10 would be expected to have a mean of cranial length greater than 190 mm.

(c) We need to find $P(180 \leq \bar{X} \leq 190)$ as follows:

$$P(180 \leq \bar{X} \leq 190) = P\left(\frac{180-185.6}{4.016} \leq \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leq \frac{190-185.6}{4.016}\right)$$
$$= P\left(\frac{180-185.6}{4.016} \leq Z \leq \frac{190-185.6}{4.016}\right)$$
$$= \phi\left(\frac{190-185.6}{4.016}\right) - \phi\left(\frac{180-185.6}{4.016}\right)$$
$$= \phi(1.10) - \phi(-1.39)$$
$$= \phi(1.10) - [1 - \phi(1.39)]$$
$$= \phi(1.10) - 1 + \phi(1.39)$$

Now, from Column A in Table 3 of the Appendix we get

$$= 0.8643 - 1 + 0.9177$$
$$= 0.7820$$

Conclusion

Thus 78.20% of the samples of size 10 would be expected to have a mean of cranial length between 180 mm and 190 mm.

(d) We need to find $P(\bar{X} = 190)$ as follows:

$P(\bar{X} = 190) = 0$ Or $P(Z = 1.10) = 0$

Conclusion

Thus 0% of the samples of size 10 would be expected to have a mean of cranial length equals to or exactly 190 mm.

Question (3)

If the mean and standard deviation of serum iron values for healthy men are 120 and 15 micrograms per 100 ml, respectively, then is what is the probability that a random sample of size 50 normal men will yield a mean:

(a) less than or equal to 125 micrograms per ml?

(b) greater than 125 micrograms per ml?

(c) between 115 and 125 micrograms per ml?

(d) equal to 125 micrograms per ml?

Solution

From the Central-limit Theorem (CLT) because we have a large random sample size ($n = 50 > 30$), the sampling distribution of the sample mean ($\bar{X}$) is approximately normal as follows:

$$\bar{X} \dot{\sim} N\left(\mu_{\bar{X}} = \mu = 120\,, \sigma_{\bar{X}}^2 = \frac{\sigma^2}{n} = \frac{(15)^2}{50} = 4.5\right)$$

$\bar{X} \dot{\sim} N(120\,, 4.5)$ is the Sampling Distribution of $\bar{X}$ approximately

with population standard deviation $\sigma_{\bar{X}} = \sqrt{\sigma_{\bar{X}}^2} = \frac{\sigma}{\sqrt{n}} = \sqrt{4.5} = 2.121$

(a) We need to find $P(\bar{X} \leq 125)$ as follows:

$$P(\bar{X} \leq 125) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leq \frac{125-120}{2.121}\right)$$
$$= \phi(2.36)$$
$$= 0.9909$$ From Column A in Table 3 of the Appendix

Conclusion

Thus 99.09% of the samples of size 50 normal men will yield a mean of serum iron less than or equal to 125 micrograms per ml.

(b) We need to find $P(\bar{X} > 125)$ as follows:

$$P(\bar{X} > 125) = 1 - P(\bar{X} \leq 125)$$
$$= 1 - P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leq \frac{125-120}{2.121}\right)$$
$$= 1 - \phi(2.36)$$
$$= 1 - 0.9909$$ From Column A in Table 3 of the Appendix
$$= 0.0091$$

Conclusion

Thus 0.91% of the samples of size 50 normal men will yield a mean of serum iron greater than 125 micrograms per ml.

(c) We need to find $P(115 \leq \bar{X} \leq 125)$ as follows:

$$P(115 \leq \bar{X} \leq 125) = P\left(\frac{115-120}{2.121} \leq \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leq \frac{125-120}{2.121}\right)$$
$$= P\left(\frac{115-120}{2.121} \leq Z \leq \frac{125-120}{2.121}\right)$$

$$= \phi\left(\frac{125-120}{2.121}\right) - \phi\left(\frac{115-120}{2.121}\right)$$
$$= \phi(2.36) - \phi(-2.36)$$
$$= \phi(2.36) - [1 - \phi(2.36)]$$
$$= 2\phi(2.36) - 1$$

Now, from Column A in Table 3 of the Appendix we get

$$= 2(0.9909) - 1$$
$$= 0.9818$$

Conclusion

Thus 98.18% of the samples of size 50 normal men will yield a mean of serum iron between 115 and 125 micrograms per ml.

(d) We need to find $P(\bar{X} = 125)$ as follows:

$P(\bar{X} = 125) = 0$ Or $P(Z = 125) = 0$

Conclusion

Thus 0% of the samples of size 50 normal men will yield a mean of serum iron equals to or exactly 125 micrograms per ml.

Exercises

Exercise (1)

If the uric acid values in normal adult males are approximately normally distributed with a mean and standard deviation of 5.7 and 1 mg percent respectively, find the probability that a sample of size 9 will yield a mean:

a. Greater than 6. **b.** Between 5 and 6.
c. Less than 5.2.

Answer: (a) 0.1841 (b) 0.1662 (c) 0.0668

Exercise (2)

For a population of 17-year-old boys, the mean subscapular skinfold thickness (in millimeters) is 9.7 and the standard deviation is 6.0. For a simple random sample of size 40 drawn from this population find the probability that the sample mean will be:

a. Greater than 11. **b.** Less than or equal to 7.5.
c. Between 7 and 10.5.

Answer: (a) 0.0853 (b) 0.0102 (c) 0.7973
